A TALE OF TWO CITIES: WHEN MATHEMATICS, COMPUTERS, AND STUDENTS MEET

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"It is a far, far better thing that we do now than we have ever done" Well, probably not quite – but we report on the extension of a project that addresses the potential of attitudinal factors to impact on situations involving the use of computers to learn mathematics. The construction of six attitude scales is discussed, together with response data from students in London and Brisbane. The data point to separate attitudinal dimensions related respectively to mathematics and to computers. This has implications for learning situations in which they interact.

BACKGROUND

Undergraduate mathematics courses in Australia have for some time been integrating symbolic algebra software into their teaching programs at an increasing rate (Pemberton, 1996). This is a local reflection of an international trend. Students are being asked to relate to computers and to mathematics in new ways, and their response to these demands forms an important focus for research. In this paper we discuss an approach to investigating attitudinal factors that emerge as consequences of interactions endemic to such an enterprise.

While the study of *attitudes* in mathematics learning has a substantial history, the relationship between *attitude* and *performance* is not clear cut although positive correlations have often been noted between these characteristics. Early claims that affective variables can predict achievement (e.g. Fennema and Sherman, 1978) have been balanced by later comments (e.g. Schoenfeld, 1989) indicating that research does not give a clear picture of the direction of causal relationships. The Tartre and Fennema (1995) comment that described *confidence* as the affective variable most consistently related to mathematics achievement is probably a safe summary of the position.

More recent studies have continued to pose the direction of the relationship between *attitude* and *performance* as an open question. Thus while Tall and Razali (1993) argued that the best way to foster positive attitudes is to provide success, Hensel and Stephens (1997) concluded that "it is still not totally clear whether achievement influences attitude, or attitude influences achievement", while Shaw and Shaw (1997) noted that among engineering undergraduates the top performing students (at entry) had a much more positive attitude to mathematics, and lower performing students a commensurately negative one – again leaving the direction of causality open.

The study of attitudes towards information technology (most frequently computers) has a shorter but more intensive history, probably because information technology, while newer, is all pervasive and permeates all curriculum areas. In fact the emphasis has been overwhelmingly towards the interaction with computers as such, rather than on the use of technology in particular learning contexts, although as Selwyn (1997) noted, awareness of students' attitudes towards computers constitutes a "central criterion in the evaluation of computer concerns and in the development of computer based curricula". Among the plethora of studies that have been conducted, Francis (1993) observed that the integration of findings is difficult because of the lack of common agreement regarding what it is that computer-related attitude scales set out to measure. In considering attitudes to information technology among graduates, studies involving mathematics students appear relatively hard to come by, although several studies have included affective variables when evaluating outcomes (see below). It is this very breadth of discipline background which has served to keep the investigation of attitudes to information technology at a general level, appropriate to the majority who will not be called upon to use computers in the same technical sense as mathematics students working intensively with specialised software.

The importance of studying attitudes to information technology in conjunction with those relating to mathematics is emphasised and re-inforced by the increasing use of technological devices in mathematics instruction, and while there have been many enthusiastic claims for the positive impact of technology on the teaching and learning of mathematics, systematic evaluations of impact are difficult to assess. Several studies refer incidentally to attitudinal impacts as well as proficiency measures and Mackie (1992) in an evaluation of computer-assisted learning in a tertiary mathematics course indicated six positive learning outcomes, three of which were related to attitudinal factors. Park (1993) in comparing a Calculus course (utilising Mathematica) with a conventionally taught program, found some improvement in disposition towards mathematics and the computer in the experimental group. However Melin-Conjeros (1992), in comparing the performance of a group of Calculus students (equipped with limited access to Derive) with a control group, noted that the attitude of both groups decreased slightly.

In particular it is not generally clear in the mathematically focused studies just which 'attitudes' have been affected by technology, as the reporting tends to be non-specific. By inference it appears that it is 'attitude' to mathematics that is referred to, and we are led to consider the implications of technology in impacting upon component attributes. The consistent and strong relationship between mathematics confidence and performance noted previously (whatever the direction of causality), means that the implications of a nexus between technology and mathematics needs specific research attention. The broad reporting of studies on the use of technology in mathematics instruction make it difficult to disentangle whether reported affective outcomes are associated with changed attitudes to mathematics, or are linked directly to the technology. Theoretically we are moved to ask about the interpretation of outcomes if students possess high mathematics confidence and motivation, but low computer confidence and motivation, and vice versa. These interests have motivated the construction of attitude measures to address such questions.

The purpose is to design attitude scales for use in programs in which computer technology is specifically directed towards assisting mathematics learning.

The context within which the scales have been designed is that involving undergraduate students newly enrolled in introductory mathematics subjects at two universities viz. City University (London), and the University of Queensland (Brisbane). The subjects involved major use of symbolic algebra packages.

THEORETICAL BASIS OF SCALE DESIGN

Given our purpose of developing scales for use in settings involving interaction between technology and mathematics learning, we now discuss the rationale behind our choice of items and methods. We have found the positions articulated by Hart (1989), Mandler (1989), and McLeod (1989, 1994) to be helpful in fashioning the approach to our definition of terms and hence instrumentation. We shall refer to this as the HMM classification in which the ordering *beliefs*, *attitudes*, *emotions* represents increasing affective involvement, decreasing cognitive involvement, and decreasing stability. Beliefs are viewed as mainly cognitive in nature being built up slowly over time, while attitude may be viewed as the end result of emotional reactions that have been internalised and automatised (McLeod, 1989) to generate feelings of moderate intensity and reasonable stability. With the decay of the emotional content with time, the response becomes more stable, and hence amenable to access through questionnaires and interviews. Emotions, as hot reactions, cannot reasonably be measured by such dispassionate means as questionnaires, and their role has been relatively underplayed in terms of research. We have adopted the HMM classification as the basis from which to develop our instrumentation. The distinction between an *attitude* and a *belief* is tenuous to a degree – we have endeavoured to seek an *attitude* focus by wording items so that the respondent is personally involved:

e.g. *I feel more confident of my answers with a computer to help me;*
rather than *Computers help people to be more confident in obtaining answers*

THE ATTITUDE SCALES

The students for whom our measures are designed are tertiary undergraduates in mathematics courses; having made this a conscious career choice-whereby mathematics has been selected as both useful in pursuing career aspirations, and as a subject compatible with themselves as individuals. Hence while we retain an overall monitoring interest, gender and usefulness, which have figured prominently in other attitude studies (e.g. Fennema and Sherman, 1976) do not play a dominant role in our design. Two of the nine attributes (confidence and motivation) represented in the Fennema-Sherman formulation have been reflected in scale development, with appropriate items constructed for use by undergraduates. The choice of these attributes was influenced strongly by the total purpose of designing instruments for use when computer technology is used in the teaching/learning context. We have chosen *confidence* and *motivation* because of their extensive appearance in the literature for both mathematics and technology, and because of their potential for discriminating between attitudes when technology and mathematics interact. These four scales are designed to measure attitudes on both dimensions so that such differences can be identified and their implications noted. In particular the choice of *confidence* and *motivation* enables two circumstances of particular interest to be identified viz situations where students hold strong positive feelings towards mathematics and negative feelings towards technology, and vice-versa. And of course *confidence* and *motivation* are two constructs that have been strongly and consistently linked with mathematics achievement over many years as discussed previously.

Two further scales address factors, important for the learning context, that are not accessed by the separate computer and mathematics scales. These are designed respectively to measure the degree of interaction between mathematics and computers that students perceive they apply in learning situations, and the degree of engagement that the students bring to their mathematical learning. The theory underlying the construction of these scales is summarised below.

The interactive significance of the learning and instructional context has been emphasised in general (e.g. McLeod 1989). Given the significance of *engagement* for effective learning (e.g. Anderson, 1995), whereby its importance is recognised by scholars as disparate as constructivists and time on task proponents, the extent to which students interact with learning materials is of central interest. In a computer environment students may simply respond to the screen or be active in note making, summarising, and experimenting. Indeed they may choose not to utilise technology when it is available and relevant (Boers and Jones, 1994). The physical separation of the learning components; pen and paper, computer screen, and human brain adds a further dimension to the co-ordinating processes required for effective learning strategies. The computer-mathematics *interaction* scale assesses the extent to which students bring their mathematical thinking into active inter-play with the computer medium.

Within each scale the (six or eight) items were arranged randomly with half requiring the reversal of polarity at the coding stage. Students were asked for a measure of their agreement (or rejection) with respect to item wording, which resulted in a 13 point Likert scale. The item groups were presented such that the constructs underlying each respective group were not known to the students. The scale items were theoretically determined from the respective underlying constructs and from cognate literature.

ADMINISTRATION & OUTCOMES

The instrument was given initially in October 1994 to 156 first year students on entry to courses in engineering, mathematics and actuarial science at City University, London, and to 56 students in a general first year mathematics subject at the University of Queensland in 1997. This represents an extension of the study described in Galbraith & Haines (1998).

The responses displayed similar patterns for both cities. To illustrate sample items we include below the items whose responses contributed most strongly to the scale score; polarities have been adjusted so that a higher score means more of the property described by the scale label. We include for each scale, the positively worded item(s) attracting the strongest support, and the negatively worded item(s) invoking the strongest rejection. (B=Brisbane, L=London). B1&L2 means that the item was the strongest choice of Brisbane students, and second strongest choice of London students.

mathematics confidence:	I can get good results in mathematics (B1&L1)
	*No matter how much I study, math is always difficult for me (B1&L1)
mathematics motivation:	If something about mathematics puzzles me, I find myself thinking about it afterwards (B1&L2)
	*If something about mathematics puzzles me I would rather someone gives me the answer than to have to work it out for myself (B1&L2)
mathematics engagement:	I find it helpful to test understanding by attempting exercises and problems (B1&L1)
	*I find working through examples less effective than memorising given material (B1&L1)
computer confidence:	I am confident I can master any computer procedure that is needed for my course (B1&L1)
	*As a male/female (*cross out that which does not apply*) I feel disadvantaged in having to use computers (B1&L1)
computer motivation:	I will work at a computer for long periods of time to successfully complete a task (B1&L2)
	* The way that computers force you to follow a procedure annoys me (B1&L2)
comp/math interaction:	I find it helpful to make notes in addition to copying material from the computer screen, or obtaining a print out (B1&L2)
	*I rarely review material soon after a computer session is finished (B1&L2)

* negatively worded item involving scale reversal

Scale Means

These are provided in Table 1 (Brisbane data in brackets).

Table 1
Scale Means

mathematics confidence	7.9 (5.6)	computer confidence	7.3 (6.7)
mathematics motivation	8.5 (6.2)	computer motivation	7.3 (6.6)
mathematics engagement	8.1 (8.4)	comp/math interaction	7.3 (6.6)

The study did not set out to compare the *level* of student response between groups. Rather interest is in the structural relationships between the mathematics and computer responses at each location. We note that the London students feel more positively about mathematics than the Brisbane students. This is to be expected, given the strong engineering representation in London versus the service course orientation of the Brisbane group. It is observed that the relative ordering of means has a similar pattern within each group, except that mathematics engagement is relatively higher among the Brisbane students.

Scale Reliabilities

These were obtained for each scale as follows-Brisbane data in brackets (see Table 2).The scales are coherent with reliabilities from strong to moderate, and with all items contributing. The bringing together of theoretically based disparate properties to address engagement and interaction issues, has unsurprisingly resulted in lower as than for more closely defined concepts like confidence and motivation.

Table 2
Scale Reliablities (Cronbach)

mathematics confidence	0.77 (0.87)	computer confidence	0.82 (0.90)
mathematics motivation	0.80 (0.86)	computer motivation	0.85 (0.91)
mathematics engagement	0.57 (0.60)	comp/math interaction	0.70 (0.69)

Scale Validity

This rests primarily upon the theoretical base behind the construction of the scales. Additional structural evidence may be inferred from the sample items given above. For example the two items attracting the strongest responses for *mathematics confidence* (expecting good results, and rejecting that mathematics is difficult irrespective of effort), are both centrally to do with confidence. The coherence of the scale as indicated in the a value then supports the argument for validity without examining each additional item. Similar arguments apply to the other scales.

DIFFERENCES IN ATTITUDES TO MATHEMATICS AND COMPUTING

A main purpose in this research is to investigate the extent to which attitudes to computer use and to mathematics represent different inputs into technology based teaching contexts involving mathematics learning. In this section we analyse the student responses to address this issue further. Consider first, correlations between the six scales (Table 3).

Table 3
Correlation Between the Attitude Scales

	mconf	mmotiv	mengag	cconf	cmotiv	cmint
mconf		.47(.81)	.08(.31)	.29(.16)	.14(.09)	.13(.14)
mmotiv			.46(.37)	.25(.41)	.29(.37)	.35(.40)
mengag				.06(.18)	.09(.04)	.26(.16)
cconf					.71(.90)	.61(.77)
cmotiv						.68(.85)
cmint						

The entries in Table 3 indicate that the confidence and motivation scales are strongly associated within mathematics (0.47,0.81) and within computing (0.71,0.90) but they are less strongly associated across the areas. This is shown by the weak correlation, for example, between mathematics confidence and computer confidence (0.29,0.16). Mathematics engagement is solidly associated (0.46,0.37) with motivation. The computer-mathematics interaction scale is more strongly associated with the computer confidence (0.61,0.77) and computer motivation (0.68,0.85) scales than with the mathematical scales. This suggests that computer attitudes are more influential than mathematical attitudes in facilitating the active engagement of computer related activities in mathematical learning. These results suggest that Factor Analysis using the six scales as input variables with a two-factor solution as goal is appropriate. Using oblimin rotation (SPSS) following a principal components analysis the loadings shown in Table 4 were obtained. The two-factor solution confirms that the computer and mathematics related scales define different dimensions with computer properties dominant in the interaction scale. Brisbane data again in brackets.

Table 4
Factor Pattern Matrix

	Factor 1	Factor 2
mconf	0.11(-0.12)	**.55(.96)**
mmotiv	0.14(.02)	**.85(.90)**
mengag	-0.17(.08)	**.79(.45)**
cconf	**.89(.92)**	-03(.05)
cmotiv	**.92(.97)**	-.05(-.09)
cmint	**.80(.89)**	.13(.07)

Percentage of variance 67.2(75.2)

REFLECTIONS

Characteristics associated with high scores on the respective scales are given below. These are based on the scale items, some of which were included in the earlier discussion. Space does not allow a more comprehensive listing. Low scoring characteristics may be inferred by reversing the positive properties noted.

Students with high mathematics confidence feel that they obtain value for effort, do not worry about new topics, expect good results, and feel generally good about mathematics as a subject.

Students with high mathematics motivation enjoy challenging mathematics, stick at problems until solved, puzzle over problems away from formal classes, become absorbed in their mathematics, and would rather work a problem out than be told the answer.

Students with high computer confidence feel comfortable about operating computers, believe they can master any computer procedure required of them, are more sure of their answers when supported by a computer, and are confident of resolving mistakes that occur during computer activity themselves.

Students with high computer motivation find using computers makes learning more enjoyable, like the freedom to experiment that a computer provides, will spend long hours at a computer, enjoy thinking up new ideas to test on a computer, and do not feel limited by the computer environment

Students with high computer-mathematics interaction feel that computers enhance mathematics learning by providing many examples, enable the user to focus on major ideas by reducing mechanical toil, use note-making to augment screen based information, review material following each computer session, and find computers helpful in linking algebraic and geometric ideas.

Students rating high on mathematics engagement prefer working examples to memorising material, like to test understanding through exercises and problems, try to link new knowledge to existing knowledge, elaborate given material with their own notes, and review work regularly.

Pedagogies to support the new focus in undergraduate teaching are still in the process of development or refinement, and within this enterprise the interaction between mathematics and technology is of significant importance. Regarding the current work, we note firstly the properties independently confirmed among students with different backgrounds in different locations. We believe this increases confidence in the robustness both of the instrument, and of the outcomes. Secondly the strong correlations between *confidence* and *motivation* within mathematics and computing respectively, suggest that one such scale might suffice if a concise instrument is envisaged. Both *mathematics engagement* and *computer/mathematics interaction* should be retained as these represent important indicators of student involvement. An issue for future tracking arises from the current work. This is whether structural differences between mathematics and computer based affective responses on attributes such as *confidence* and *motivation* will diminish with time, or whether they represent distinctive sets of characteristics with a permanent presence in computer assisted mathematics learning. That this issue is a real one was confirmed by the 34 Brisbane students who responded to an invitation to answer the open ended question "How do you feel about using computers to learn mathematics?" There were 15 positive responses, 14 negative responses, and five that contained both positive and negative comments!

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